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Leptonic and Semileptonic Decays of Pseudoscalar Mesons

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Abstract

We employ the relativistic constituent quark model to give a unified description of the leptonic and semileptonic decays of pseudoscalar mesons (π, K, D, D_s, B, B_s). The calculated leptonic decay constants and form factors are found to be in good agreement with available experimental data and other approaches. We reproduce the results of spin-flavor symmetry in the heavy quark limit.

1 Introduction

Semileptonic decays of pseudoscalar mesons allow to evaluate the elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, which are fundamental parameters of the Standard Model. The decay $K \to \pi e \nu$ provides the most accurate determination of V_{us}, the semileptonic decays of D and B mesons, $D \to K(K^*)l\nu$, $B \to D(D^*)l\nu$ and $B \to \pi(\rho)l\nu$, can be used to determine $|V_{cs}|$, $|V_{cb}|$ and $|V_{ub}|$, respectively. The effects of strong interactions in these processes can be expressed in terms of form factors, which depend on q^2, the squared momentum transferred to the leptonic pair. Information on the form factors are obtained by measuring the distributions of q^2 and decay angles.

The decays of heavy D and B mesons are of particular interest due to the spin-flavor symmetry observed for infinite quark masses [1]. This symmetry allows

to reduce the number of form factors and express them in terms of the universal Isgur-Wise function [2]. Also the scaling laws derived for some physical observables can be, in principle, tested experimentally. Since the Isgur-Wise function cannot be calculated from first principles, many models and non-perturbative approaches, which exhibit the heavy quark symmetry, have been employed to describe relevant phenomena. However, it was found out, that the finite mass corrections are very important, especially, in the charm sector. It appears that in some sense a step back should be done from using the heavy quark symmetry as a guide under model building to the straightforward calculations with full quark propagators. Then one has to check the consistency of the results with the spin-flavor symmetry in the heavy quark limit.

In this paper we employ the relativistic constituent quark model (RCQM) [3] for the simultaneous description of both light and heavy flavored meson leptonic and semileptonic decays. This model is based on the effective Lagrangian describing the coupling of mesons with their quark constituents, and the compositeness condition. The physical processes are described by the one-loop quark diagrams with free constituent propagators and meson-quark vertices related to the Bethe-Salpeter amplitudes. The masses of lower-lying pseudoscalar (PS) mesons should be less than the sum of quark constituent masses to provide the absence of imaginary parts corresponding to quark production. The adjustable parameters, the widths of Bethe-Salpeter amplitudes in momentum space and constituent quark masses, are determined from the best fit of available experimental data and some lattice determinations. We found that our results are in good agreement with experimental data and other approaches. Also we reproduce the results of spin-flavor symmetry for leptonic decay constants and semileptonic form factors in the heavy quark limit.

The shapes of vertex functions and quark propagators should be found from the Bethe-Salpeter and Dyson-Schwinger equations, respectively. This is provided by the Dyson-Schwinger Equation (DSE) [4] studies. A DSE-approach has been employed to provide a unified and uniformly accurate description of light- and heavy-meson observables [5,6].

A similar approach, based on the effective heavy meson Lagrangian, has been described in [7] in terms of a model based on meson-quark interactions, where mesonic transition amplitudes are represented by diagrams with heavy mesons attached to quark loops. The free propagator has been used for light quarks. However, the quark propagator obtained in the heavy quark limit has been employed for heavy quarks.

2 The model

We employ an approach [3] based on the effective interaction Lagrangian which describes the transition of hadron into quarks. For example, the transition of the meson H into its constituents q_1 and q_2 is given by the Lagrangian

$$\mathcal{L}_{\rm int}(x) = g_H H(x) \int dx_1 \int dx_2 \Phi_H(x; x_1, x_2) \bar{q}(x_1) \Gamma_H \lambda_H q(x_2)\,. \tag{1}$$

Here, λ_H and Γ_H are the Gell-Mann and Dirac matrices, respectively, which provide the flavor and spin numbers of the meson H. The function Φ_H is related to the scalar part of Bethe-Salpeter amplitude. For instance, the separable form $\Phi_H(x; x_1, x_2) = \delta(x - (x_1 + x_2)/2) f((x_1 - x_2)^2)$ has been used in [3] for pions.

The coupling constants g_H is given by the so called *compositeness condition* proposed in [8] and extensively used in [9]. That condition means that the renormalization constant of the meson field is equal to zero:

$$Z_H = 1 - \frac{3g_H^2}{4\pi^2} \tilde{\Pi}'_H(m_H^2) = 0\,, \tag{2}$$

where $\tilde{\Pi}'_H$ is the derivative of the meson mass operator defined by

$$\tilde{\Pi}_H(p^2) = \int \frac{d^4k}{4\pi^2 i} \phi_H^2(-k^2) \mathrm{tr}\Big[\Gamma_H S_2(\not k) \Gamma_H S_1(\not k + \not p)\Big]. \tag{3}$$

The invariant amplitudes describing the leptonic $H(p) \to l\nu$ and semileptonic $H(p) \to H'(p') l\nu$ decays are written down

$$A(H(p) \to e\nu) = \frac{G_F}{\sqrt{2}} V_{qq'} (\bar{e} O_\mu \nu) M_H^\mu(p) \tag{4}$$

$$A(H(p) \to H'(p') e\nu) = \frac{G_F}{\sqrt{2}} V_{qq'} (\bar{e} O_\mu \nu) M_{HH'}^\mu(p, p'), \tag{5}$$

where G_F is the Fermi weak-decay constant, $V_{qq'}$ is the appropriate element of the CKM matrix. The matrix elements of the hadronic currents are given by

$$M_H^\mu(p) = \frac{3}{4\pi^2} g_H \int \frac{d^4k}{4\pi^2 i} \phi_H(-k^2) \mathrm{tr}\Big[\gamma^5 S_2(\not k) O^\mu S_1(\not k + \not p)\Big] = f_H p^\mu \tag{6}$$

$$
\begin{aligned}
M^{\mu}_{HH'}(p,p') = & \frac{3}{4\pi^2} g_H g_{H'} \int \frac{d^4k}{4\pi^2 i} \phi_H(-k^2)\phi_{H'}(-k^2) \qquad (7) \\
& \times \mathrm{tr}\Big[\gamma^5 S_3(\not k)\gamma^5 S_2(\not k+\not p')O^{\mu} S_1(\not k+\not p)\Big] \\
& = f_+(q^2)(p+p')^{\mu} + f_-(q^2)(p-p')^{\mu} \qquad (8)
\end{aligned}
$$

where $\phi_H(-k^2)$ is related to the BS-amplitude in momentum space, and

$$
S_i(\not k) = \frac{1}{m_i - \not k} \qquad (9)
$$

is the propagator of the constituent quark with mass m_i. As discussed before, to avoid the appearance of imaginary parts in Eqs. (6) and (7), we assume that $m_H < m_{q_1} + m_{q_2}$ which is a reliable approximation for the lower-lying mesons considered here.

To evaluate the integral in Eq. (7)

$$
I_{HH'}(p,p') = \int \frac{d^4k}{4\pi^2 i} \mathcal{F}(-k^2)\mathrm{tr}\Big\{\gamma^5 S_3(\not k)\gamma^5 S_2(\not k+\not p')\gamma^{\mu} S_1(\not k+\not p)\Big\}, \qquad (10)
$$

where $\mathcal{F}(-k^2) = \phi_H(-k^2)\cdot\phi_{H'}(-k^2)$, we need to calculate the following integrals:

$$
J^{(0,\mu,\mu\nu,\mu\nu\delta)} = \int \frac{d^4k}{\pi^2 i} \frac{(1, k^{\mu}, k^{\mu}k^{\nu}, k^{\mu}k^{\nu}k^{\delta})\mathcal{F}(-k^2)}{[m_1^2-(k+p)^2][m_2^2-(k+p')^2][m_3^2-k^2]}\,. \qquad (11)
$$

Using the Cauchy representation for the function $\mathcal{F}(-k^2)$ and then the standard techniques of the Feynman $\alpha-$parametrization one finds ($\mathcal{F}'(z) = d\mathcal{F}(z)/dz$)

$$
J^0 = \int_0^{\infty} dt \Big(\frac{t}{1+t}\Big)^2 \int d^3\alpha\, \delta\Big(1-\sum_{i=1}^{3}\alpha_i\Big)\Big(-\mathcal{F}'(z_I)\Big) \qquad (12)
$$

$$
J^{\mu} = -\int_0^{\infty} dt \Big(\frac{t}{1+t}\Big)^3 \int d^3\alpha\, \delta\Big(1-\sum_{i=1}^{3}\alpha_i\Big) P^{\mu}_{\alpha}\Big(-\mathcal{F}'(z_I)\Big) \qquad (13)
$$

$$
J^{\mu\nu} = \int_0^{\infty} dt \Big(\frac{t}{1+t}\Big)^2 \int d^3\alpha\, \delta\Big(1-\sum_{i=1}^{3}\alpha_i\Big) \qquad (14)
$$

$$\times\Big\{-\frac{1}{2}g^{\mu\nu}\frac{1}{1+t}\mathcal{F}(z_I)-P_\alpha^\mu P_\alpha^\nu\Big(\frac{t}{1+t}\Big)^2\mathcal{F}'(z_I)\Big\}$$

$$J^{\mu\nu\delta}=\int\limits_0^\infty dt\Big(\frac{t}{1+t}\Big)^2\int d^3\alpha\,\delta\Big(1-\sum_{i=1}^3\alpha_i\Big) \tag{15}$$
$$\times\Big\{\frac{1}{2}\Big[g^{\mu\nu}P_\alpha^\delta+g^{\mu\delta}P_\alpha^\nu+g^{\nu\delta}P_\alpha^\mu\Big]\frac{t}{(1+t)^2}\mathcal{F}(z_I)$$
$$+P_\alpha^\mu P_\alpha^\nu P_\alpha^\delta\Big(\frac{t}{1+t}\Big)^3\mathcal{F}'(z_I)\Big\}$$

where $q=p-p'$, $P_\alpha=\alpha_1 p+\alpha_2 p'$, $D_3=\alpha_1\alpha_3 p^2+\alpha_2\alpha_3 p'^2+\alpha_1\alpha_2 q^2$, and $z_I=t[\sum_{i=1}^3\alpha_i m_i^2-D_3]-P_\alpha^2 t/(1+t)$.

Finally, Eq. (10) becomes

$$I^\mu_{HH'}(p,p')=(p+p')^\mu\,I_+(p^2,p'^2,q^2)+(p-p')^\mu\,I_-(p^2,p'^2,q^2)$$

with

$$I_+(p^2,p'^2,q^2)=\frac{1}{2}\int\limits_0^\infty dt\Big(\frac{t}{1+t}\Big)^2\int d^3\alpha\,\delta\Big(1-\sum_{i=1}^3\alpha_i\Big) \tag{16}$$
$$\times\Big\{\mathcal{F}(z_I)\frac{1}{1+t}\Big[4-3(\alpha_1+\alpha_2)\frac{t}{1+t}\Big]$$
$$-\mathcal{F}'(z_I)\Big[(m_1+m_2)m_3$$
$$+\frac{t}{1+t}\Big(-(\alpha_1+\alpha_2)(m_1m_3+m_2m_3-m_1m_2)$$
$$+\alpha_1p^2+\alpha_2p'^2\Big)-P_\alpha^2\Big(\frac{t}{1+t}\Big)^2\Big(2-(\alpha_1+\alpha_2)\frac{t}{1+t}\Big)\Big]\Big\}.$$

The normalization condition is written in the form

$$\frac{3g_H^2}{4\pi^2}I_+(p^2,p^2,0)=1 \tag{17}$$

with $m_1=m_2\equiv m$.

The integrals corresponding to the matrix element of the leptonic decay $H(p)\to l\nu$ and radiative decay of neutral meson $H(p)\to\gamma(q_1)+\gamma(q_2)$ are calculated following the same procedure. We have

$$Y^\mu(p) = \int \frac{d^4k}{4\pi^2 i}\phi(-k^2)\mathrm{tr}\Big\{\gamma^5 S_2(\not k)\gamma^\mu(I-\gamma^5)S_1(\not k+\not p)\Big\} = p^\mu Y(p^2)$$

$$Y(p^2) = \int\limits_0^\infty dt \frac{t}{(1+t)^2}\int\limits_0^1 d\alpha\Big[m_2 + (m_1-m_2)\frac{\alpha t}{1+t}\Big]\phi(z_Y) \qquad (18)$$

$$K^{\mu\nu}(q_1,q_2) = \int \frac{d^4k}{4\pi^2 i}\phi(-k^2)\mathrm{tr}\Big\{\gamma^5 S(\not k-\not q_2)\gamma^\mu S(\not k)\gamma^\nu S(\not k+\not q_1)\Big\}$$
$$= i\varepsilon^{\mu\nu\alpha\beta}q_1^\alpha q_2^\beta K(p^2)$$

$$K(p^2) = m\int\limits_0^\infty dt\Big(\frac{t}{1+t}\Big)^2\int\limits_0^1 d\alpha_1\int\limits_0^{1-\alpha_1} d\alpha_2\Big(-\phi'(z_K)\Big) \qquad (19)$$

where $z_Y = t[\alpha m_1^2 + (1-\alpha)m_2^2 - \alpha p^2 + \alpha^2 p^2 t/(1+t)]$ and $z_K = t[m_1^2 - \alpha_1\alpha_2 p^2] + \alpha_1\alpha_2 p^2 t/(1+t)$.

The physical observables are expressed in terms of the structural integrals written in Eqs. (16), (18) and (19):

$$g_{P\gamma\gamma} = \frac{g_P}{2\sqrt{2}\pi^2}K(m_P^2), \qquad \Gamma(P\to\gamma\gamma) = \frac{\pi}{4}\alpha^2 m_P^3 g_{P\gamma\gamma}^2, \qquad (20)$$

$$f_P = \frac{3}{4\pi^2}\, g_P\, Y(m_P^2), \quad \Gamma(P\to l\nu) = |V_{qq'}|^2\frac{G_F^2 f_P^2}{8\pi}m_P m_l^2\Big[1-\frac{m_l^2}{m_P^2}\Big]^2, \quad (21)$$

$$f_+(q^2) = \frac{3}{4\pi^2}\, g_P g_{P'}\, I_+(m_P^2, m_{P'}^2, q^2), \qquad (22)$$

$$\Gamma(P\to P'l\nu) = |V_{qq'}|^2\frac{G_F^2}{192\pi^3 m_P^3}\int\limits_0^{t_-} dt|f_+(t)|^2\Big[(t_+-t)(t_--t)\Big]^{3/2},$$

with $t_\pm = (m_P \pm m_{P'})^2$ (the extra factor 1/2 appears for π^0 in the final state).

2.1 Heavy quark limit

The leptonic heavy decay constants and semileptonic heavy to heavy form factors acquire a simple form in the heavy quark limit, *i. e.* when $m_1 \equiv M \to \infty$, $m_2 \equiv M' \to \infty$ and $p^2 = (M+E)^2$, $p'^2 = (M'+E)^2$ with E being a constant value. From Eq. (16) by replacing the variables $\alpha_1 \to \alpha_1/M$ and $\alpha_2 \to \alpha_2/M'$, one obtains

$$I_{+} \to \frac{M+M'}{2MM'} \cdot \int\limits_0^\infty dt \Big(\frac{t}{1+t}\Big)^2 \int\limits_0^1 d\alpha\alpha \int\limits_0^1 d\tau \Big(-\mathcal{F}'(z)\Big)\Big[m + \frac{\alpha t}{1+t}\Big]$$

$$= \frac{M+M'}{2MM'} \cdot \frac{1}{2} \int\limits_0^1 \frac{d\tau}{W} \int\limits_0^\infty du \mathcal{F}(\tilde{z}) \frac{m+\sqrt{u}}{m^2+\tilde{z}} \tag{23}$$

where $\tilde{z} = u - 2E\sqrt{u/W}$, $W = 1 + 2\tau(1-\tau)(w-1)$ and $w = (M^2 + M'^2 - 2MM'q^2)/(2MM')$.

The normalization condition can be obtained from Eq. (23) by putting $w = 1$ and $M' = M$. We have

$$\frac{3g_H^2}{4\pi^2} \cdot I_{+}^{(0)} = 1, \qquad I_{+}^{(0)} = \frac{1}{2M} I_N, \qquad I_N = \int\limits_0^\infty du \phi_H^2(\tilde{z}_0) \frac{m+\sqrt{u}}{m^2+\tilde{z}_0} \tag{24}$$

where $\tilde{z}_0 = u - 2E\sqrt{u}$. Then the leptonic decay constant and semileptonic form factors are written as

$$f_P \to \frac{1}{\sqrt{M}} \cdot \sqrt{\frac{3}{2\pi^2 I_N}} \int\limits_0^\infty du [\sqrt{u} - E] \phi_H(\tilde{z}_0) \frac{m + \sqrt{u}/2}{m^2 + \tilde{z}_0} \tag{25}$$

$$f_{\pm} \to \frac{M' \pm M}{2\sqrt{MM'}} \xi(w) \qquad \xi(w) = \frac{1}{I_N} \int\limits_0^1 \frac{d\tau}{W} \int\limits_0^\infty du \phi_H^2(\tilde{z}) \frac{m+\sqrt{u}}{m^2+\tilde{z}} . \tag{26}$$

It is readily seen that we reproduce the scaling law for both leptonic decay constants and form factors, and obtain the explicit expression for the Isgur-Wise function [1,2].

3 Results and discussion

The expressions obtained in the previous section for physical observables are valid for any vertex function $\phi_H(-k^2)$. Here, we choose a Gaussian form $\phi(-k^2) = \exp\{k^2/\Lambda_H^2\}$ in Minkowski space. The magnitude of Λ_H characterizes the size of the BS-amplitude and is an adjustable parameter in our approach. Thus, we have six Λ-parameters plus the four quark masses, all of which are fixed via the least-squares fit to the observables measured experimentally or taken from lattice simulations (see, Table 1).

The fit yields the values of Λ-parameters and the constituent quark masses

Table 1
Calculated values of a range of observables ($g_{\pi\gamma\gamma}$ in GeV^{-1}, leptonic decay constants in GeV, form factors and ratios are dimensionless). The "Obs." are extracted from Refs. [10–16]. The quantities used in fitting our parameters are marked by "$*$".

		Obs.	Calc.			Obs.	Calc.
$*$	$g_{\pi\gamma\gamma}$	0.274	0.242		$f_+^{K\pi}(0)$	0.98	0.98
$*$	f_π	0.131	0.131	$*$	$f_+^{DK}(0)$	0.74 ± 0.03	0.74
$*$	f_K	0.160	0.160		$f_+^{BD}(0)$		0.73
$*$	f_D	0.191^{+19}_{-28}	0.191		$f_+^{B\pi}(0)$	0.27 ± 0.11	0.51
$*$	$\frac{f_{D_s}}{f_D}$	1.08(8)	1.08		$\mathrm{Br}(K \to \pi l \nu)$	$(4.82 \pm 0.06) \cdot 10^{-2}$	$4.4 \cdot 10^{-2}$
	f_{D_s}	0.206^{+18}_{-28}	0.206		$\mathrm{Br}(D \to K l \nu)$	$(6.8 \pm 0.8) \cdot 10^{-2}$	$8.1 \cdot 10^{-2}$
$*$	f_B	0.172^{+27}_{-31}	0.172		$\mathrm{Br}(B \to D l \nu)$	$(2.00 \pm 0.25) \cdot 10^{-2}$	$2.3 \cdot 10^{-2}$
$*$	$\frac{f_{B_s}}{f_B}$	1.14(8)	1.14		$\mathrm{Br}(B \to \pi l \nu)$	$(1.8 \pm 0.6) \cdot 10^{-4}$	$2.1 \cdot 10^{-4}$
	f_{B_s}		0.196				

which are listed in Eqs. (27) and (28).

Λ_π	Λ_K	Λ_D	Λ_{D_s}	Λ_B	Λ_{B_s}	(in GeV)
1.16	1.82	1.87	1.95	2.16	2.27	

(27)

m_u	m_s	m_c	m_b	(in GeV)
0.235	0.333	1.67	5.06	

(28)

The values of Λ are such that $\Lambda_{m_i} < \Lambda_{m_j}$ if $m_i < m_j$. This corresponds to the ordering law for sizes of bound states. The values of $\Lambda_D = 1.87$ GeV and $\Lambda_B = 2.16$ GeV are larger than those obtained in [6]: $\Lambda_D = 1.41$ GeV and $\Lambda_B = 1.65$ GeV. The mass of u-quark and the parameter Λ_π are almost fixed from the decays $\pi \to \mu\nu$ and $\pi^0 \to \gamma\gamma$ with an accuracy of a few percent. The obtained value of the u-quark mass $m_u = 0.235$ GeV is less than the constituent-light-quark mass typically employed in quark models for baryon physics ($m_u > m_N/3 = 0.313$ GeV). For instance, the value of $m_u = 0.420$ GeV was extracted from fitting nucleon observables within our approach [3]. The different choice of constituent quark masses is a common feature of quark models with free propagators due to the lack of confinement. However, we consider here the low-lying mesons that allows us to fix the constituent quark masses in a self-consistent manner. As mentioned above, the meson masses must be less than the sum of masses of their constituents. This gives the restrictions on the choice of the meson binding energies: $E_K = m_K - m_s < m_u$, $E_D = m_D - m_c < m_u$ and $E_B = m_B - m_b < m_u$, which means that the

binding energy cannot be relatively large as compared with those obtained in [6]: $E_D = 0.58$ GeV and $E_B = 0.74$ GeV.

Let us now consider the q^2-behaviour of the form factors. We use the three-parameter function for the four f_+ form factors

$$f_+^{HH'}(q^2) = \frac{f(0)}{1 - b_0(q^2/m_H^2) - b_1(q^2/m_H^2)^2} \tag{29}$$

here b_0 , b_1 and $f(0)$ are parameters to be fitted. We collect the fitted values in the following Table and report the q^2-dependence in Fig. 1.

	$K \to \pi$	$D \to K$	$B \to D$	$B \to \pi$
b_0	0.28	0.64	0.77	0.52
b_1	0.057	0.20	0.19	0.38

(30)

For comparison, we plot, together with our results, the predictions of a vector dominance monopole model:

$$f_+^{q\to q'}(q^2) = \frac{f_+^{q\to q'}(0)}{1 - q^2/m^2_{V_{qq'}}} \tag{31}$$

with $m^2_{V_{qq'}}$ being a mass of lower-lying $\bar{q}q'$-vector meson. We choose $m_{D_s^*} = 2.11$ GeV for $c \to s$, $m_{B^*} = 5.325$ GeV for $b \to u$, $m_{B_c^*} \approx m_{B_c} = 6.4$ GeV [17] for $b \to c$ transitions. The values of $f_+^{qq'}(0)$ are taken from the Table 1. Also we calculate the branching ratios of semileptonic decays by using widely accepted values of the CKM matrix elements [10].

Our result for the slope of the K_{l3} form factor

$$\lambda_+ = m_\pi^2 \frac{f_+^{K\pi\prime}(0)}{f_+^{K\pi}(0)} = 0.023 \; , \tag{32}$$

is in good agreement with experiment: $\lambda_+^{\rm expt} = 0.0286 \pm 0.0022$ [10] and VDM prediction: $\lambda_+^{\rm VDM} = m_\pi^2/m_{K^*}^2 = 0.025$. This value is also consistent with Refs. [18]

One can see that the agreement with experimental data and lattice results is very good, with the exception of the value of $f_+^{bu}(0)$ which is found to be larger than the monopole extrapolation of a lattice simulation, QCD Sum Rules (cf. [19]) and some other quark models (see, for example, [20,21]). However, this

result is consistent with the value calculated from Refs. [6,22] and allows us to reproduce the experimental data for $B \to \pi l \nu$ with quite good accuracy.

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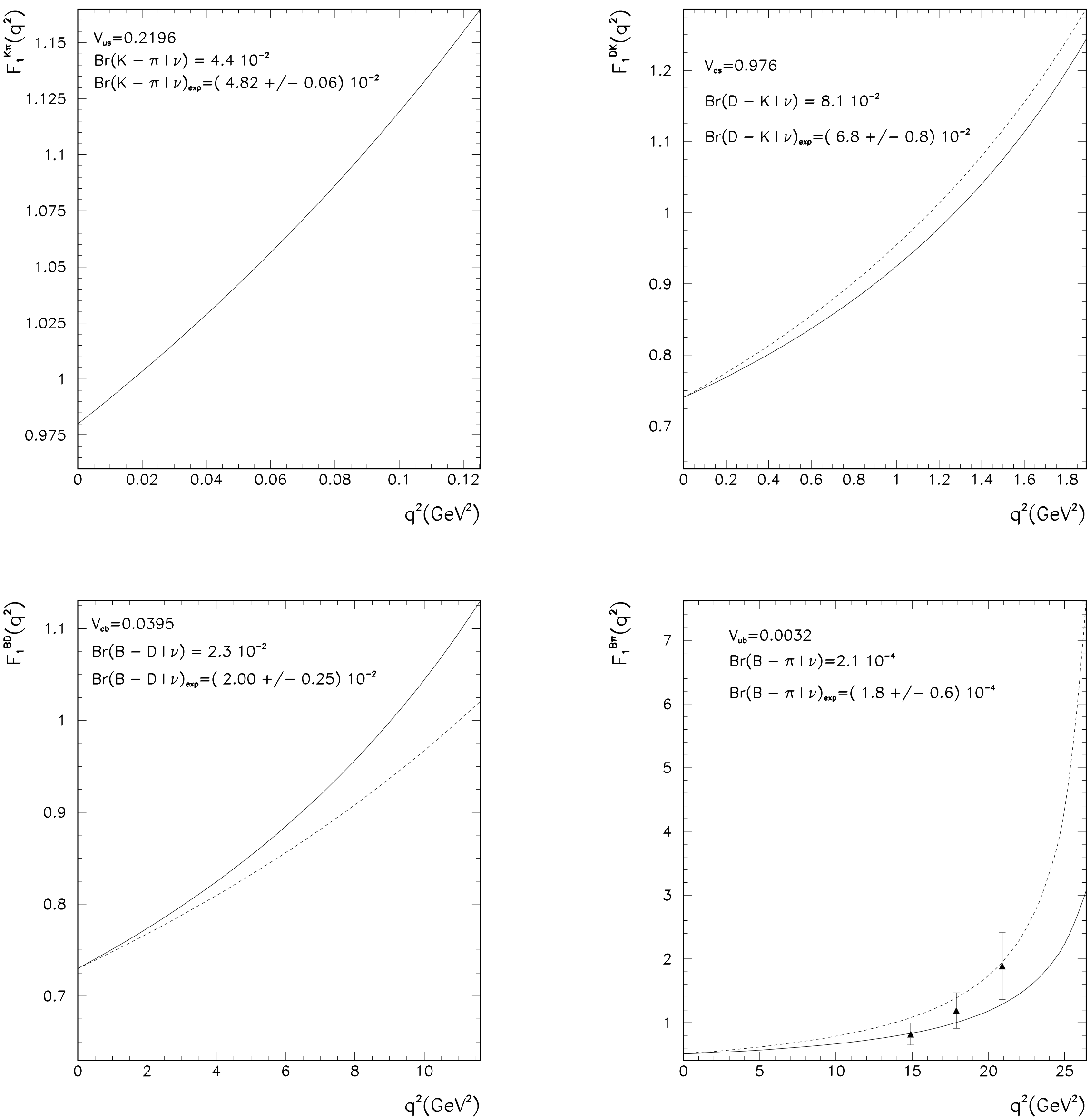

Fig. 1. The semileptonic $K \to \pi$, $D \to K$, $B \to D$ and $B \to \pi$ form factors with, for comparison, a vector dominance, monopole model Eq. (31) and a lattice simulation [23]. Our results: continuous lines. Monopole: dotted lines. Lattice: data points.